Lösungsvorschlag zu Grundaufgaben zur ln-Funktion

Nr		BE

7.1

$f(x) = x^2 \cdot \ln x\ , \qquad D_f = \mathbb{R}^+$

NSt.: $x_1 = 0 \notin D_f\ ,\ \ x_2 = 1$ d.h. eine Nullstelle $x = 1$

$$\lim_{\substack{x\to 0\\ x>0}} x^2 \cdot \ln x = „+0 \cdot -\infty" = \lim_{\substack{x\to 0\\ x>0}} \frac{\ln x}{\frac{1}{x^2}} = \quad (l'H.) \lim_{\substack{x\to 0\\ x>0}} \frac{1}{x\cdot\left(-\frac{2}{x^3}\right)} = \lim_{\substack{x\to 0\\ x>0}} -\frac{x^2}{2} = -0$$

$$\lim_{x\to\infty} x^2 \cdot \ln x = „+\infty \cdot +\infty" = +\infty$$

7.2

$f'(x) = 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x} = 2x \cdot \ln(x) + x = x \cdot (2\ln(x) + 1)$

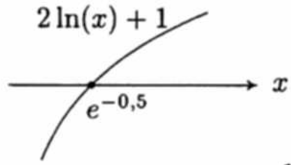
$f''(x) = 1 \cdot (2\ln(x) + 1) + x \cdot \frac{2}{x} = 2\ln(x) + 3$

7.3

Monotonie: $f'(x) = 0: \quad x \cdot (2\ln(x) + 1) = 0 \quad (x > 0) \iff$

$$2\ln(x) + 1 = 0 \iff \ln x = -\tfrac{1}{2} \iff x = e^{-0,5} = \frac{1}{\sqrt{e}}\ , \quad f\left(\frac{1}{\sqrt{e}}\right) = \tfrac{1}{e}\cdot\left(-\tfrac{1}{2}\right) = -\frac{1}{2e}$$

$f'(x) = x \cdot (2\ln(x) + 1) \lesseqgtr 0 \iff x \lesseqgtr \frac{1}{\sqrt{e}}\ ,\ \ x \in D_f$



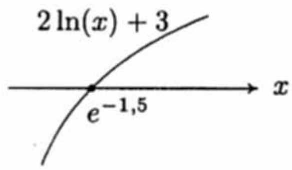
$\Longrightarrow f$ streng monoton abnehmend in $]0; \frac{1}{\sqrt{e}}]$ und streng monoton zunehmend in $[\frac{1}{\sqrt{e}}; \infty[$

$\Longrightarrow T(\frac{1}{\sqrt{e}} | -\frac{1}{2e})$ Tiefpunkt von G_f

7.4

Krümmung: $f''(x) = 0: \quad \ln(x) = -\frac{3}{2} \iff x = e^{-1,5}\ , \quad f(e^{-1,5}) = (e^{-1,5})^2 \cdot (-\frac{3}{2}) = -\frac{3}{2e^3}$

$f''(x) = 2\ln(x) + 3 \lesseqgtr 0 \iff x \lesseqgtr e^{-1,5}\ ,\ \ x \in D_f$



$\Longrightarrow G_f$ rechtsgekrümmt in $]0; e^{-1,5}]$ und linksgekrümmt in $[e^{-1,5}; \infty[$

$\Longrightarrow W(e^{-1,5} | -\frac{3}{2e^3})$ Wendepunkt von G_f

7.5